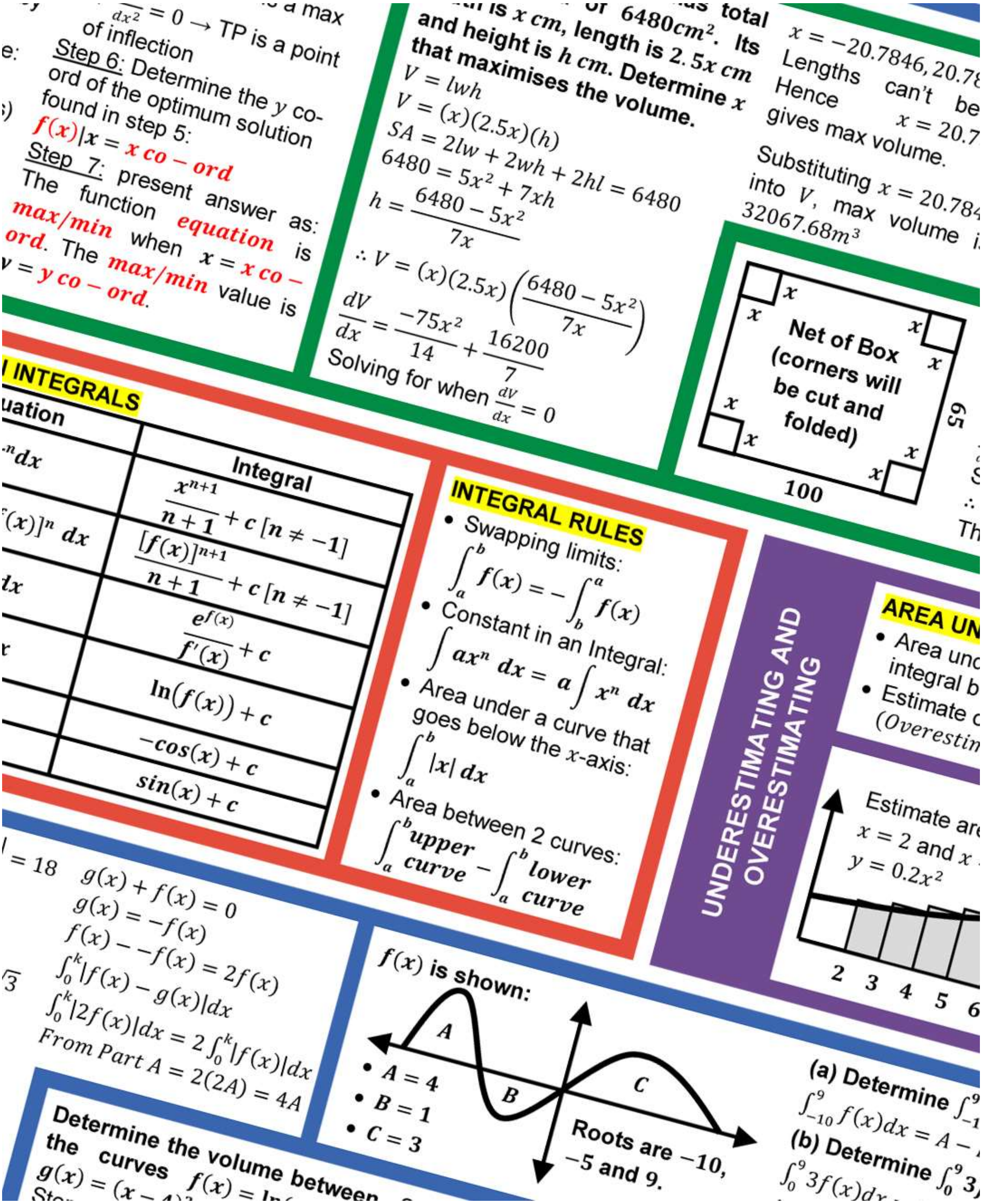

ATAR Mathematics Methods Units 3 & 4

Exam Notes for WA Year 12 Students

Created by Anthony Bochrinis
Version 1.3 (22/09/17)

Sharpened®

Year 12 ATAR Mathematics Methods Units 3 & 4 Exam Notes

About the Creator – Anthony Bochrinis

I graduated from high school in 2012, completed a Bachelor of Actuarial Science in 2015 and am currently completing my Graduate Diploma in Secondary Education with the goal of becoming a full-time high school teacher next year!

My original exam notes (created in 2013) were inspired by Severus Snape's copy of Advanced Potion Making in Harry Potter and the Half-Blood Prince; a textbook filled with annotations containing all of the pro tips and secrets to help gain a clearer understanding.

I hope that my exam notes help to sharpen your knowledge and I wish you all the best in your exams!

Using these Exam Notes
These exam notes are designed to be a complement to your studies throughout the year. As such, I recommend using these exam notes during class, during tests, whilst studying at home or in the library and even in the calculator-assumed section of your mock and WACE exams.

These exam notes contain theory, diagrams, formulae and worked examples based off the official SCSA syllabus to give you a full revision of the entire course in just 4 pages. For more detailed information about our most frequently asked questions about the use of these exam notes, please visit my website or email me.

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... without further ado, I present my exam notes!

INDICES / SURDS / LOGARITHMS / EULER'S NUMBER

RULES & LAWS

INDEX LAWS

$a^m \times a^n = a^{m+n}$	$a^m \div a^n = a^{m-n}$
$a^0 = 1$	$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$
$(a^m)^n = a^{m\times n}$	$(ab)^m = a^m b^m$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$a^{-m} = \frac{1}{a^m}$

SURD LAWS

$\sqrt{a}\times\sqrt{b} = \sqrt{ab}$	$\sqrt{a}\times\sqrt{a} = a$
$\sqrt{a}\div\sqrt{b} = \sqrt{\frac{a}{b}}$	$m\sqrt{a} \pm n\sqrt{a} = (m\pm n)\sqrt{a}$

RATIONALISING A SURD

$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}\times 1 = \frac{1}{\sqrt{a}}\times\frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$

LOGARITHM LAWS

$a^x = y \rightarrow x = log_a(y)$
$e^x = y \rightarrow x = log_e(y)$
$x = log_e(y) \rightarrow x = \ln(y)$
$log_a(1) = 0 \rightarrow a^0 = 1$
$log_a(a) = 1 \rightarrow a^1 = a$

$log_a(xy) = log_a(x) + log_a(y)$
$log_a\left(\frac{x}{y}\right) = log_a(x) - log_a(y)$
$log_a(x)^n = n\,log_a(x)$

CHANGING LOG BASE (B TO A)

$log_a(x) = \frac{log_b(x)}{log_b(a)}$

EULER'S NUMBER

$e = 2.718281828...$

EXPONENTIAL LAWS

$\lim_{h\to 0}\left(\frac{a^h - 1}{h}\right) = \ln(a)$

$\lim_{h\to 0}\left(\frac{e^h - 1}{h}\right) = 1$

GRAPHING LOGARITHMS

LOGARITHM FORMULA

$y = log_a(x - b) + c$

For logarithm curves, there is a condition that $a > 1$ whereas b and c can take on any value.

Log graphs have a vertical asymptote at $x = b$.

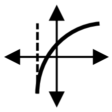
When $b = 0$ and $c = 0$, the x-intercept of the curve is at (1,0).

$a \rightarrow 1$	Makes curve steeper
$a \rightarrow \infty$	Makes curve shallower
$b > 0$	Shift curve right by b
$b < 0$	Shift curve left by b
$c > 0$	Shift curve up by c
$c < 0$	Shift curve down by c

$y = log_2(x - a) + b$ is below. Find a and b.

x-intercept @ $(-1.5, 0)$

Vertical Asymptote @ $x = -2$



The vertical asymptote @ $x = -2$ means that $a = -2$

$\therefore\ y = log_2(x + 2) + b$

$0 = log_2(-1.5 + 2) + b$

$-b = log_2(0.5) \Longrightarrow b = 1$

$\therefore y = log_2(x + 2) + 1$

Given $y = log_a(x - 3)$ and $(84, 4)$ is a point on the line, find the value of a.

$y = log_a(x - 3)$

$4 = log_a(84 - 3)$

$4 = log_a(81)$

$a = 3$

LOGARITHM QUESTIONS

Solve the equation for x: $log_2(x) + log_2(x - 2) = 3$

$log_2(x^2 - 2x) = 3$

$x^2 - 2x = 2^3 = 8$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4\ or\ -2$

Let $x = log_5(2)$ and $y = log_5(3)$. In terms of x and y, find:

(a) $log_5(18)$

$= log_5(2\times 3^2)$

$= log_5(2) + 2log_5(3)$

$= x + 2y$

(b) $log_5(60)$

$= log_5(5\times 2^2\times 3)$

$= log_5(5) + 2log_5(2) + log_5(3) = 1 + 2x + y$

(c) $log_8(3)$

$= \frac{log_5(3)}{log_5(8)} = \frac{log_5(3)}{log_5(2^3)} = \frac{y}{3x}$

Let $A = 50log_{10}\left(\frac{B}{35}\right)$. How many times greater is B if A is 150 than if A is 250?

When A is 150:

Solve: $150 = 50log_{10}\left(\frac{B}{35}\right)$

$B = 35{,}000$

When A is 250:

Solve: $250 = 50log_{10}\left(\frac{B}{35}\right)$

$B = 3{,}500{,}000$

Determining the Factor:

$= \frac{B_{A=250}}{B_{A=150}} = \frac{3500000}{35000} = 100$

If $Y = log(X)$, find the ratio of $A{:}B$ if A is Y when X is 84000 and B is Y when X is 21000000.

$A{:}B = \frac{B}{A}$

$= \frac{log(21000000)}{log(84000)}$

$= \frac{7.3222}{4.9242}$

$= 1.4870$

$\therefore A{:}B = 1{:}1.4870$

APPLICATIONS OF CALCULUS

DERIVATIVE RULES & LAWS

EXPRESSING DERIVATIVES

Type	1st Derivative	2nd Derivative
$y = \cdots$	$\frac{dy}{dx} = \cdots$	$\frac{d^2y}{dx^2} = \cdots$
$f(x) = \cdots$	$f'(x) = \cdots$	$f''(x) = \cdots$

NATURE OF TURNING POINTS

Type	$f'(x)$	$f''(x)$
Minimum (Convex)	0	+
Maximum (Concave)	0	–
Horizontal Inflection Point	0	0
Vertical Inflection Point	+ –	0

DIFFERENTIATION RULES

Type	Equation	1st Derivative
Product Rule	$y = uv$	$\frac{dy}{dx} = u'v + uv'$
Quotient Rule	$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$
Chain Rule	$y = [f(x)]^n$	$\frac{dy}{dx} = n[f(x)]^{n-1}\times f'(x)$

INFLECTION TYPES

- A vertical point of inflection has a gradient (1st derivative) not equal to 0 and has a 2nd derivative equal to 0. Shown on the right:
- A horizontal point of inflection has a gradient (1st derivative) of 0 and also has a 2nd derivative equal to 0. Shown on the right:

RATES OF CHANGE

- Instantaneous rate of change @ $time = t$: $f'(t)$
- Average rate of change between a and b: $\frac{f(b)-f(a)}{b-a}$
- Net change in $f(x)$ between a and b: $\int_a^b |f'(x)|dx$

FINANCE FORMULAE

- Profit Formula: $P(x) = R(x) - C(x)$
- Revenue Formula: $R(x) = p(x)q(x)$
- Average Cost: $\frac{C(x)}{x}$

COMMON DERIVATIVES

Equation	1st Derivative
$y = ax^n$	$\frac{dy}{dx} = n\times ax^{n-1}$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)\times e^{f(x)}$
$y = \frac{1}{x} = x^{-1}$	$\frac{dy}{dx} = \frac{-1}{x^2} = -x^{-2}$
$y = \pm sin(x)$	$\frac{dy}{dx} = \pm cos(x)$
$y = \pm cos(x)$	$\frac{dy}{dx} = \mp sin(x)$
$y = \ln[f(x)]$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^x$	$\frac{dy}{dx} = \ln(a)\times a^x$

CLASSPAD DERIVATIVES

FIND POINT WITH A GIVEN GRADIENT

$diff(f(x))|x = x\ co-ord$

Where:
- **diff** is found in Main App → Action → Calculation
- **|** is found in Keyboard → Math3
- $x\ co-ord$ is the x co-ordinate of the point given.

FIND GRADIENT OF A POINT

$solve(diff(f(x))) = gradient$

$f(ans)$

Where:
- **solve** is found in Main App → Action → Advanced
- 1st answer gives the x co-ord and 2nd answer gives y co-ord.

EQUATION OF THE TANGENT

$tanLine(f(x), x, x\ co-ord)$

Where:
- **tanLine** is found in Main App → Calculation → line
- Answer is the linear equation of the tangent in the form of $y = mx + c$

CO-ORDS OF A TURNING POINT

$solve(diff(f(x))) = 0$

$f(ans)$

Where:
- **solve** is found in Main App → Action → Advanced
- 1st answer gives the x co-ord and 2nd answer gives y co-ord.

SECOND DERIVATIVE AT A POINT

$diff(diff(f(x)))$

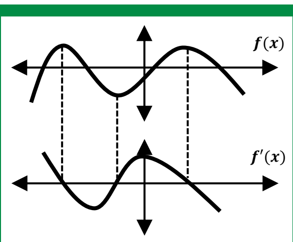
$|x = x\ co-ord$

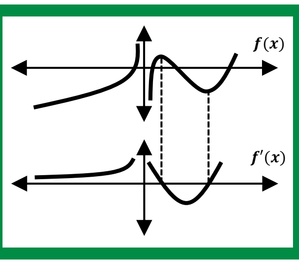
Where:
- Answer gives the value of the second derivative at the point.

SKETCHING DERIVATIVES

SKETCHING DERIVATIVES

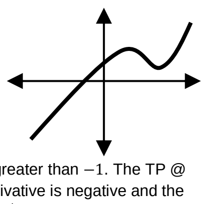
- All local maximum and minimums are x-intercepts on the derivative.
- Where the function is increasing, the derivative is above the x-axis.
- Where the function is decreasing, the derivative is below the x-axis.
- Where there is a point of inflection on the graph (vertical or horizontal), the derivative has a maximum or minimum turning point.





Sketch a graph and its derivative with the following features:

- **$y \geq 0$ for $x \geq -1$**
- **$dy/dx = 0$ for $x = 1, 2$**
- **$d^2y/dx^2 > 0$ for $x = 2$**
- **$d^2y/dx^2 < 0$ for $x = 1$**



y is positive for all x that is greater than -1. The TP @ $x = 1$ is a max as the 2nd derivative is negative and the TP @ $x = 2$ is a min as the 2nd derivative is positive.

DERIVATIVE QUESTIONS

Determine the equation of the tangent to $f(x) = \ln(5) + x^2e$ at the point $f(x) = e$.

Step 1: Find $f'(x)$

$f'(x) = 2ex$

Step 2: sub $x = e$ into $f'(x)$

$f'(e) = 2e(e) = 2e^2$

$2e^2 = m$ in $y = mc + c$

Step 3: Determine the y co-ord when $x = e$

$f(e) = \ln(5) + e^3$

Step 4: Solve for c in $y = mx + c$, substituting in values for y, m and x

$y = mx + c$

$\ln(5) + e^3 = 2e^2(e) + c$

$c = \ln(5) - e^3$

Step 5: present the equation of the tangent

$y = mx + c$

$m = 2e^2$

$c = \ln(5) - e^3$

$y = 2xe^2 - e^3 + \ln(5)$

For $f(x) = \frac{cos(x)}{2x+2}$, find $f'(\pi)$.

$f'(x) = \frac{(-sin(x))(2x+2) - (cos(x))(2)}{(2x+2)^2}$

$f'(x) = \frac{2(-sin(x))(x+1) - 2cos(x)}{(2x+2)^2}$

$f'(\pi) = \frac{1}{2(\pi+1)^2}$

Determine the co-ordinates of the second maximum point of $f(x) = 2sin(3x)$ for $0 \leq x \leq \pi$

$f'(x) = 6cos(3x)$. Solving for $x = 0$:

$x = 0.5236, 1.5708, 2.6180$

$f''(x) = -18sin(3x)$, using 2nd derivative test, 2nd max @ $(2.6180, 2)$

INCREMENTAL FORMULA

Find the approximate change in y when x changes from 3 to 2.98 in the equation: $y = 3x^2 - 2x$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$y = 3x^2 - 2x$

$\frac{dy}{dx} = 6x - 2$

$\delta y \approx \frac{dy}{dx}\times\delta x$

$\delta y \approx (6x - 2)\times\delta x$

$\delta y \approx (6(3) - 2)\times(-0.02)$

$\delta y \approx -0.32\ \therefore$ decrease by 0.32

The radius of a sphere increases from $15cm$ to $15.1cm$, what is the approx increase in surface area?

$\frac{\delta S}{\delta r} \approx \frac{dS}{dr}$

$S = 4\pi r^2$

$\frac{dS}{dr} = 8\pi r$

$\delta S \approx \frac{dS}{dr}\times\delta r$

$\delta S \approx (8\pi r)\times\delta r$

$\delta S \approx (8\pi(15))\times(0.01)$

$\delta S \approx 3.77\ \therefore$ increase by $3.77cm^2$

Find the approximate change in y when x changes from 1 to 1.1 in the equation: $y = sin(2x) + e^{3x}$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$y = sin(2x) + e^{3x}$

$\frac{dy}{dx} = 2\cos(2x) + 3e^{3x}$

$\delta y \approx \frac{dy}{dx}\times\delta x$

$\delta y \approx (2\cos(2x) + 3e^{3x})\times\delta x$

$\delta y \approx 2\cos(2(1)) + 3e^{3(1)}\times(0.1)$

$\delta y \approx 5.94\ \therefore$ increase by 5.94

The radius of a sphere increases by 2%, find the % increase in the volume.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{\delta V}{\delta r} \approx \frac{dV}{dr}$

$\delta V \approx \frac{dV}{dr}\times\delta r$

$\delta V \approx 4\pi r^2\times\delta r$

$\frac{\delta V}{V} \approx \frac{4\pi r^2\delta r}{V}$

$\frac{\delta V}{V} \approx \frac{4\pi r^2\delta r}{4\pi r^3/3}$

$\frac{\delta V}{V} \approx \frac{3\delta r}{r}$

$\frac{\delta V}{V} \approx 3\times\frac{\delta r}{r}$

$\frac{\delta V}{V} \approx 3\times 2\%$

$\frac{\delta V}{V} \approx 3\times 6\%$

$\therefore$ increase by 6%

APPLICATIONS OF CALCULUS

GROWTH AND DECAY

GROWTH & DECAY
- $A = A_0 e^{kt}$
- $\frac{dA}{dt} = kA_0 e^{kt} = kA$

Where:
- $A_0 \rightarrow$ Initial (starting) amount
- $k \rightarrow$ continuous rate of change
- $A \rightarrow$ amount @ time t

What is the half-life of the following equation that tracks radioactivity of a substance: $A = 800e^{-0.04t}$?

$A = 800e^{-0.04t}$
$400 = 800e^{-0.04t}$
$t = 17.33\ days$

HALF LIFE
Time taken for amount to reduce by 50% (i.e. $A = 0.5A_0$).

DOUBLING TIME
Time taken for amount to increase by 100% (i.e. $A = 2A_0$).

(a) The temperature of the south pole over t years has the equation $T = A + Be^{-kt}$. The initial temp is $-50°C$ and the long term temp is $-20°C$, find A and B.

$-50 = A + Be^{-k(0)} \rightarrow -50 = A + B$
$-30 = A + Be^{-k(\infty)}$
$A = -30$ and $B = -20$

(b) Determine k if after 10 years the temperature is $-43.5°C$

$-43.5 = -30 - 20e^{-k(10)} \rightarrow k = 0.0393$

(a) The population of a small island is $P = 40 - 10e^{-kt}$, show that $dP/dt = k(P - 40)$.

$\frac{dP}{dt} = 10ke^{-kt}$ & $P = 40 - 10e^{-kt}$
Hence, $\frac{dP}{dt} = (-P + 40) \times k$
$\frac{dP}{dt} = k(P - 40)$

(b) If $k = 0.02$, find the rate at which the pop is changing when the pop is 30.

$\frac{dP}{dt} = 0.02(30 - 40) = -0.2$

(a) If $dA/dt = 0.252A$ is an exponential equation, find the initial value for A given that A @ $time = 10$ is 565.

$A = A_0 e^{kt} \rightarrow 565 = A_0 e^{0.252(10)}$
$565/A_0 = A_0 e^{0.252(10)}$
$ln(565/A_0) = 0.252(10)$
$ln(565) - ln(A_0) = 2.52$
$ln(A_0) = ln(565) - 2.52 = 3.82$
$A_0 = e^{3.82} = 45.60$

OPTIMISATION

OPTIMISATION STEPS

Step 1: Draw a diagram of the scenario and define all variables.
Step 2: Only 2 variables can be used to optimise a problem, if there are more than 2 variables, reduce the number of variables by substitution and simplification.
Step 3: Determine the derivative: $diff(f(x))$
Step 4: Determine the x co-ord(s) of the turning point(s): $solve(diff(f(x))) = 0$
Step 5: Determine the nature of the turning point(s): $diff(diff(f(x)))|x = x\ co - ord$

Note: repeat step 5 for each x co-ord found in step 4.
- If $\frac{d^2y}{dx^2} > 0 \rightarrow$ TP is a min
- If $\frac{d^2y}{dx^2} < 0 \rightarrow$ TP is a max
- If $\frac{d^2y}{dx^2} = 0 \rightarrow$ TP is a point of inflection

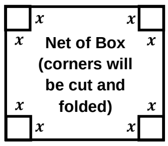
Step 6: Determine the y co-ord of the optimum solution found in step 5: $f(x)|x = x\ co - ord$
Step 7: present answer as: The function ***equation*** is ***max/min*** when $x = x\ co - ord$. The ***max/min*** value is $y = y\ co - ord$.

A rectangular prism has total surface area of $6480cm^2$. Its width is $x\ cm$, length is $2.5x\ cm$ and height is $h\ cm$. Determine x that maximises the volume.

$V = lwh$
$V = (x)(2.5x)(h)$
$SA = 2lw + 2wh + 2hl = 6480$
$6480 = 5x^2 + 7xh$
$h = \frac{6480 - 5x^2}{7x}$
$\therefore V = (x)(2.5x)\left(\frac{6480 - 5x^2}{7x}\right)$
$\frac{dV}{dx} = \frac{-75x^2}{14} + \frac{16200}{7}$
Solving for when $\frac{dV}{dx} = 0$

$x = -20.7846, 20.7846$
Lengths can't be 0. Hence $x = 20.7846$ gives max volume.

Substituting $x = 20.7846$ into V, max volume is $32067.68m^3$



A rectangular box is to be made from a sheet of metal with squares of length x to be cut from the corners. If the sheet of metal is 65cm wide and 100cm long, determine the value of x that will maximise the volume of the box.

$V = lwh \rightarrow$ There are 4 variables in this equation, we need to eliminate 2 variables by substitution:
$l = 100 - 2x$, $w = 65 - 2x$ and $h = x$
$V = (100 - 2x)(65 - 2x)x = 4x^3 - 330x^2 + 6500x$
$\frac{dV}{dx} = 12x^2 - 660x + 6500$
Solving for when $\frac{dV}{dx} = 0$: $x = 12.85, 42.15$
$\frac{d^2V}{dx^2}@x = 12.85 = -351.6\ \therefore maximum$
$\frac{d^2V}{dx^2}@x = 42.15 = 351.6\ \therefore minimum$
Sub $x = 12.85$ to find max $V = 37522cm^3$
$\therefore$ The volume is maximised when $x = 12.85cm$.
The maximum volume is $37522cm^3$.

INTEGRATION RULES & LAWS

INDEFINITE INTEGRAL
- $\int x\, dx$

The answer will be an equation and remember to put a '+c' at the end.

DEFINITE INTEGRAL
- $\int_a^b x\, dx$

Where:
- a – lower bound
- b – upper bound

The answer will be a single number (all other variables are eliminated).

COMMON INTEGRALS

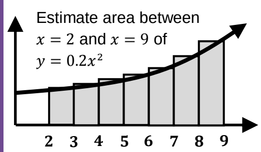
Equation	Integral
$\int x^n dx$	$\frac{x^{n+1}}{n+1} + c\ [n \neq -1]$
$\int f'(x) \times [f(x)]^n\ dx$	$\frac{[f(x)]^{n+1}}{n+1} + c\ [n \neq -1]$
$\int e^{f(x)}\ dx$	$\frac{e^{f(x)}}{f'(x)} + c$
$\int \frac{f'(x)}{f(x)}\ dx$	$\ln(f(x)) + c$
$\int sin(x)\ dx$	$-cos(x) + c$
$\int cos(x)\ dx$	$sin(x) + c$

INTEGRAL RULES
- Swapping limits: $\int_a^b f(x) = -\int_b^a f(x)$
- Constant in an Integral: $\int ax^n\ dx = a\int x^n\ dx$
- Area under a curve that goes below the x-axis: $\int_a^b |x|\ dx$
- Area between 2 curves: $\int_a^b upper\ curve - \int_a^b lower\ curve$

UNDERESTIMATING AND OVERESTIMATING

AREA UNDER A CURVE USING RECTANGLES
- Area under a curve is determined by the definite integral but can be estimated using rectangles.
- Estimate of area between $x = a$ and $x = b$ is: $(Overestimate + Underestimate)/2$

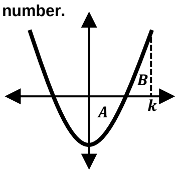
Estimate area between $x = 2$ and $x = 9$ of $y = 0.2x^2$



Overestimate:
$0.2(2)^2 + \cdots + 0.2(9)^2 = 56.8$
Underestimate:
$0.2(1)^2 + \cdots + 0.2(8)^2 = 40.8$
Estimate Area:
$\frac{56.8 + 40.8}{2} = 48.8$

DEFINITE AND INDEFINITE INTEGRALS

(a) Part of the curve $f(x) = x^2 - 3$ is shown below. A value of k exists such that the area of the region marked A is equal to the area of the region marked B. Determine the value of k as an exact number.



Area A = $\left|\int_0^3 x^2 - 9\ dx\right| = 18$
Area B = $\int_3^k x^2 - 9\ dx$
$\therefore \int_3^k x^2 - 9\ dx = 18$
Solving for k gives $k = 3\sqrt{3}$

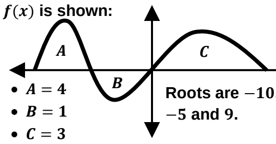
(b) Define $\int_0^k |f(x)|dx$ in terms of A.

$\int_0^k |f(x)|dx = A + B$
$But\ A = B\ so\ \int_0^k |f(x)|dx = 2A$

(c) $g(x)$ is another function such that $g(x) + f(x) = 0$. Use this to show that: $\int_0^k |f(x) - g(x)|dx = 4A$.

$g(x) + f(x) = 0$
$g(x) = -f(x)$
$f(x) - -f(x) = 2f(x)$
$\int_0^k |f(x) - g(x)|dx$
$\int_0^k |2f(x)|dx = 2\int_0^k |f(x)|dx$
$From\ Part\ A = 2(2A) = 4A$

$f(x)$ is shown:



- $A = 4$
- $B = 1$
- $C = 3$

Roots are -10, -5 and 9.

(a) Determine $\int_{-10}^{9} f(x)dx$
$\int_{-10}^{9} f(x)dx = A - B + c = 6$

(b) Determine $\int_0^9 3f(x)dx$
$\int_0^9 3f(x)dx = 3 \times C = 9$

(c) Determine $\int_9^{-5} f(x)dx$
$= -\int_{-5}^{9} f(x)dx = -C + B = -2$

(d) Determine $\int_{-10}^{-5} f(x) - 2dx$
$= \int_{-10}^{-5} f(x)dx - \int_{-10}^{-5} 2dx$
$= A - [2x]_{-10}^{-5}$
$= 4 - 10 = -6$

Determine the volume between the curves $f(x) = \ln(x)$ and $g(x) = (x - 4)^2$

Step 1: Determine the points of intersection between the two curves by solving:
$f(x) = g(x) \rightarrow \ln(x) = (x - 4)^2$
$x = 2.96, 5.29$

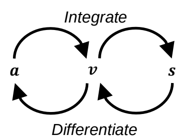
Step 2: pick a number between the two solved x values (e.g. 4) and substitute it into both equations to determine the upper function.
$f(4) = 1.39$ and $g(4) = 0$
Hence $f(x)$ is the upper curve and $g(x)$ is the lower curve.

Step 3: determine the integral that calculates the area between two curves using:
$\int_a^b upper\ curve - \int_a^b lower\ curve$
$= \int_a^b f(x)dx - \int_a^b g(x)dx$
$= \int_{2.96}^{5.29} \ln(x)\,dx - \int_{2.96}^{5.29} (x - 4)^2 dx = 2.18$

Note: the limits are the same for both integrals.

RECTILINEAR MOTION

RELATIONSHIP BETWEEN TYPES OF MOTION



Where:
- $a \rightarrow$ acceleration
- $v \rightarrow$ velocity
- $s \rightarrow$ displacement

RECTILINEAR RULES
- Change in displacement between times a and b: $\int_a^b v\ dt$
- Distance travelled between times a and b: $\int_a^b |v|\ dt$
- Object changes direction whenever $v = 0$
- Object returns to the starting position whenever $s = 0$

(a) Displacement of a body is given by $s = -t^3 + at^2 + bt + 3$ where t is the time in seconds. The body is temporarily at rest when $t = 1$. The initial velocity is 5 m/s. What is a and b?

$v = \frac{d}{dt}(-t^3 + at^2 + bt + 3)$
$v = -3t^2 + 2at + b$
When $t = 0, v = 5$
$\therefore b = 5\ \&\ v = -3t^2 + 2at + 5$
$0 = -3(1)^2 + 2a(1) + 5$
$\therefore a = -1$

(b) Find the formula for acceleration.

$a = \frac{d}{dt}(v)$
$a = \frac{d}{dt}(-3t^2 - 2t + 5)$
$a = -6t - 2$

(a) Acceleration of a body is $a = 49 - 8t$ where motion is measures in m/s. After 5 seconds the particle is instantaneously stationary. Find the formula for velocity.

$v = \int a\ dt$
$v = \int 49 - 8t\ dt$
$v = 49t - 4t^2 + c$
$0 = 49(5) - 4(5)^2 + c$
Solve for c: $c = -145$
$\therefore v = 59t - 4t^2 - 145$

(b) Find the distance travelled in the first 10 seconds.

$= \int_0^{10} |59t - 4t^2 - 145|\ dt$
$= 578.13$ metres

A body has initial displacement of $10m$ and velocity $v = t^2 + 3t$. Find the displacement when it has velocity of $63m/s$?

Solve $63 = t^2 + 3t \rightarrow t = ~~-9.58~~, 6.58$
$s = \int v\ dt = \int t^2 + 3t\ dt = \frac{t^3}{3} + \frac{3t^2}{2} + c$
$10 = \frac{(0)^3}{3} + \frac{3(0)^2}{2} + c \rightarrow c = 10$
$\therefore s = \frac{t^3}{3} + \frac{3t^2}{2} + 10$ and $s(6.58) = 169.9m$

FUNDAMENTAL THEOREM OF CALCULUS

FUNDAMENTAL THEOREM OF CALCULUS
- $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$
- $\int_a^b f'(x)dx = f(b) - f(a)$

QUESTIONS WITH FUNCTIONS AS LIMITS

Step 1: sub the limits into t.
Step 2: multiply the answer by the derivative of the limit.
(Note: for questions with 2 limits, do steps 1 and 2 twice).

$\frac{d}{dx}\left(\int_0^x t^2 dt\right) = x^2$

$\frac{d}{dx}\left(\int_0^x \ln(t)dt\right) = \ln(x)$

$\frac{d}{dx}\left(\int_0^x e^{2t}dt\right) = e^{2x}$

$\frac{d}{dx}\left(\int_0^x \sqrt{t}dt\right) = \sqrt{x}$

$\frac{d}{dx}\left(\int_0^{3x^2} \frac{1 + t}{2 - t}dt\right)$

Sub $3x^2$ into t:
$= \frac{1 + 3x^2}{2 - 3x^2}$
Multiply by the derivative of $3x^2$:
$= 6x\left(\frac{1 + 3x^2}{2 - 3x^2}\right)$
$= \frac{6x + 18x^3}{2 - 3x^2}$

$\frac{d}{dx}\left(\int_{sin(x)}^{x^3} \sqrt{t^2 + 1}\ dt\right)$

Sub x^3 into t:
$= \sqrt{(x^3)^2 + 1}$
$= \sqrt{x^6 + 1}$
Multiply by the derivative of x^3:
$= 3x^2\sqrt{x^6 + 1}$
Repeat steps 1 and 2:

sub $sin(x)$ into t:
$= \sqrt{(sin(x))^2 + 1}$
Multiply by the derivative of $sin(x)$:
$= cos(x)\sqrt{(sin(x))^2 + 1}$
Subtract the 2nd answer from the 1st answer:
$= 3x^2\sqrt{x^6 + 1}$
$- cos(x)\sqrt{(sin(x))^2 + 1}$

Determine $f(x)$ with the following conditions:
- $F(x) = \int_0^x f(t)dt$
- $\frac{d^2F}{dx^2} = x + 5$
- $F(3) = 5.$

$\frac{dF}{dx} = f(x)$
Hence $\frac{d^2F}{dx^2} = f'^{(x)} = x + 5$
Integrating $f'(x)$ to get $f(x)$:
$f(x) = \int x + 5dx = \frac{x^2}{2} + 5x + c$
As $\frac{dF}{dx} = f(x)$, we can use the

$F(x)$ formula to solve for c:
$F(3) = \int_0^3 f(t)dt = 5$
$5 = \int_0^3 \frac{t^2}{2} + 5t + c\ dt$
Using ClassPad to solve:
$c = -7.33$
$\therefore f(x) = \frac{t^2}{2} + 5t - 7.33$

Tip: If you are getting stuck on these harder questions, break down all of the derivatives given in terms of $f(x)$ and dy/dx.

PROBABILITY RULES & LAWS

PROBABILITY NOTATION
- $\cup \rightarrow$ Union (or)
- $\cap \rightarrow$ Intersection (and)
- $| \rightarrow$ given
- $\acute{A}$ or $\bar{A} \rightarrow$ complement
- $\emptyset \rightarrow$ null set
- $\in \rightarrow$ element of
- $\subset \rightarrow$ subset

PROBABILITY RULES
- $0 \leq P(A) \leq 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$ or $P(B|A) = \frac{P(A \cap B)}{P(A)}$
- $P(\bar{A}) = P(\acute{A}) = 1 - P(A)$
- For independent events: $P(A \cap B) = P(A) \times P(B)$

EXPECTED VALUE AND VARIANCE

EXPECTED VALUE AND VARIANCE
- $E(X) = \mu =$ Expected Value
- $Var(X) =$ Variance
- $\sqrt{Var(X)} =$ S.D.

Comparing $E(X)$ and $Var(X)$
- $Var(X) = E(X)^2 - [E(X)]^2$

EFFECTS OF LINEAR CHANGE
If X is a random variable and $Y = aX + b$ then:
- $E(X) = aE(X) + b$
- $Var(X) = a^2 Var(X)$

Where a and b are constants.

If $E(X) = 5$ and $Var(X) = 2$

(a) Determine $E(X + 11)$
$= E(X) + 11 = 5 + 11 = 16$

(b) Determine $E(1 - 2x)$
$= 1 - 2E(X) = 1 - (2 \times 5) = -9$

(c) Determine $Var(3X + 1)$
$= 3^2 Var(X) = 9 \times 2 = 18$

DISCRETE RANDOM VARIABLES (DRV)

ABOUT DRV
- Discrete distributions cover events that can be counted.
- It is only measured in integers (whole numbers).
- For example, counting how many students there are in each class of a school.

DRV TYPES
- Bernoulli Distribution
- Binomial Distribution

DRV RULES
- $\sum p(x) = 1$
- $0 < p(x) < 1$
- $E(X) = \sum_i p_i x_i$
- $Var(X) = \sum_i p_i (x_i - \mu)^2$

DRV QUESTIONS

(a) Is the following distribution discrete?

x	-1	0	1	2
$p(x)$	0.3	0.2	0.1	0.4

Yes, as all probabilities add to 1.

(b) Is the following distribution discrete?

x	0	1	2	3
$p(x)$	-0.1	0	0.5	0.6

No, as $p(x)$ cannot be negative.

(a) Determine the values of a and b in the following discrete distribution if $E(X) = 0.20$

x	0	1	2	3	4
$p(x)$	0.85	0.12	a	b	0.005

Equation 1: $0.12 + 2a + 3b + 0.2 = 0.2$
Equation 2: $0.85 + 0.12 + a + b + 0.005 = 1$
ClassPad simultaneous solve:
$a = 0.015$ and $b = 0.01$

BERNOULLI DISTRIBUTION

ABOUT BERNOULLI
- A Bernoulli trial is a binomial distribution with 1 trial.
- A Bernoulli trial has only two possible outcomes, which we may term "success" or "failure."
- E.g. tossing a coin is a Bernoulli trial: you can either get heads or tails.

BERNOULLI NOTATION
$X \sim B(p)$ where:
- p – probability of success

BERNOULLI RULES
- $p(x) = \begin{cases} p & for\ x = 1 \\ 1 - p & for\ x = 0 \end{cases}$
- $E(X) = p$
- $Var(X) = p(1 - p)$
- $\sigma = \sqrt{p(1 - p)}$

BINOMIAL DISTRIBUTION

ABOUT BINOMIAL
- A Binomial distribution is when you perform more than 1 independent Bernoulli trial.
- The Binomial distribution counts the number of success in an experiment with trials.
- For example, tossing a coin repeat times and counting the number of heads flipped.

BINOMIAL NOTATION
$X \sim B(n, p)$ where:
- n – number of trials
- p – probability of success

BINOMIAL RULES
- $p(x) = \binom{n}{x} (p)^x (1 - p)^{n-x}$
- $E(X) = np$
- $Var(X) = np(1 - p)$
- $\sigma = \sqrt{np(1 - p)}$

BERNOULLI VS. BINOMIAL

BERNOULLI VS. BINOMIAL
- If the number of trials is equal to 1, the distribution is Bernoulli.
- If the number of trials is more than 1, the distribution is Binomial.

CLASSPAD BERNOULLI

CLASSPAD BERNOULLI
To find:
- $p(x)$
- $P(a \leq x \leq b)$
- the value of k given $P(X \leq k)$

Use the Binomial Distribution commands to the right and set all instances of n to 1.

CLASSPAD BINOMIAL

FIND $P(X)$
$binomialPDF(x, n, p)$
Where:
- **binomialPDF** is found in Main App → Action → Distribution → Discrete
- x is the number of successful trials.

FIND $P(A \leq X \leq B)$
$binomialCDF(A, B, n, p)$
Where:
- **binomialCDF** is found in Main App → Action → Distribution → Continuous
- A is the lower bound.
- B is the upper bound.

FIND K GIVEN $P(X \leq K)$
$invBinomialCDF(P(X \leq k), n, p)$
Where:
- **invBinomialCDF** is found in Main App → Action → Distribution → Inverse
- n is the number of trials.
- p is the probability of success.

BBINOMIAL GRAPHS

ANALYSING GRAPHS
A graph is suited for Binomial if it is either negatively skewed (long left tail) or positively skewed (long right tail).

As n increases, graphs become more symmetrical (normal distribution).

BERNOULLI & BINOMIAL QUESTIONS

(a) Find the probability that a student passes a multi-choice test with 10 questions and 4 options per question by guessing answers?
$X \sim B(10, 0.25)$
$P(X \geq 5) = P(5 \leq X \leq 10) = 0.0781$

(b) If the class has 15 students, what is the probability that at least 4 pass by guessing?
$X \sim B(15, 0.0781)$
$P(X \geq 4) = P(4 \leq X \leq 15) = 0.0252$

(a) The chance of an apple being rotten in a delivery is 0.1. Find the probability that of 6 apples, 1 is rotten.
$= 0.9^5 \times 0.1 = 0.0590$

(b) Find the probability that exactly one of six apples chosen from the box are found to be rotten.
$X \sim B(6, 0.1)$
$= P(X = 1) = 0.3543$

(a) X is a binomial variable. Determine the value of parameters n and p if $E(X) = 21$ and $Var(X) = 6.3$.
$E(X) = 21 = np$ & $Var(x) = 6.3 = np(1 - p)$
ClassPad simultaneous solve:
$n = 30$ and $p = 0.7$

(b) Determine $P(X \geq 10 | X \leq 15)$
$X \sim B(30, 0.7)$
$= \frac{P(X \geq 10 \cap X \leq 15)}{P(X \leq 15)} = \frac{P(10 \leq X \leq 15)}{P(X \leq 15)}$
$= 0.9996$

The probability of a successful trial is 0.4, how many trials are needed to ensure that the probability of 3 or more successes is exceeds 0.75?
$X \sim B(n, 0.4)$ and $P(X \geq 3) > 0.75$

Method 1:
Using trial and error for different values of n on ClassPad:
$binomialCDF(3, \infty, n, 0.4) \rightarrow$ when $n = 9, CDF = 0.7682\ \therefore 9$

Method 2:
$P(X \geq 4) > 0.75 = P(X = 0) + P(X = 1) + P(X = 2) > 0.75$
$= \binom{n}{0}(0.6)^n + \binom{n}{1}(0.4)(0.6)^{n-1} + \binom{n}{2}(0.4)^2(0.6)^{n-2} = 0.75$
Solving for n on ClassPad, $n \approx 9$

CONTINUOUS RANDOM VARIABLES (CRV)

ABOUT CRV
- CRV's cover events that can be measured.
- Measures with decimal values (exact numbers).
- E.g measuring height of people in a city in cm.

CRV TYPES
- Uniform Distribution
- Normal Distribution

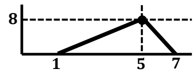
CRV RULES
- $\int p(x)dx = 1$
- $f(x) \geq 0$
- $P(X > a) = P(X \geq a)$
- $P(X = a) = 0$
- $P(a \leq X \leq b) = \int_a^b p(x)dx$
- $E(X) = \int_{-\infty}^{\infty} xp(x)dx$
- $Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx$ where $\mu = E(X)$

CRV QUESTIONS

X is a CRV. It is known that $P(X > 5) = 0.6$ and X has a probability density function of:
$$f(x) = \begin{cases} ax + b & 0 \leq x \leq 10 \\ 0 & elsewhere \end{cases}$$
Determine the values of a and b.
Equation 1: $\int_0^{10} ax + bdx = 1$
Equation 2: $\int_5^{10} ax + bdx = 0.6$
ClassPad simultaneous solve:
$a = 0.008$ and $b = 0.06$

Y is a CRV and has a probability density function of:
$$f(y) = \begin{cases} 2y^2 + 3 & 0 \leq x \leq 2 \\ 0 & elsewhere \end{cases}$$
Determine $E(Y)$ and $Var(Y)$.
$E(Y) = \int_0^2 (y)(2y^2 + 3)dy$
$E(Y) = 14$
$Var(Y) = \int_0^2 (y - 14)^2 (2y^2 + 3)dy$
$Var(Y) = 1850.1333$

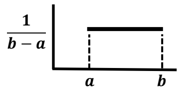
Z is a CRV and is graphed on the set of axes below:



Determine $f(z)$.
$$f(z) = \begin{cases} 2x - 2 & 1 \leq x \leq 5 \\ -4x + 28 & 5 \leq x \leq 7 \\ 0 & elsewhere \end{cases}$$

UNIFORM DISTRIBUTION

ABOUT UNIFORM
- A Uniform distribution has constant probability.
- E.g. a volcano erupts every hour. You arrive there at random and wait 30 minutes, what is the chance it erupts?



UNIFORM NOTATION
$X \sim U(a, b)$ where:
- a – lower bound
- b – upper bound

UNIFORM RULES
- $f(x) = \frac{1}{b-a}$
- $E(X) = \frac{1}{2}(a + b)$
- $Var(X) = \frac{1}{12}(b - a)^2$
- $\sigma = \sqrt{\frac{1}{12}(b - a)^2}$
- $p(x) = 0$

UNIFORM CLASSPAD

FIND $P(C \leq X \leq D)$
$\int_C^D \frac{1}{b - a} dx$

FIND $P(X \geq C | X \leq D)$
$\frac{\int_C^D \frac{1}{b-a} dx}{\int_A^D \frac{1}{b-a} dx}$

FIND K GIVEN $P(X \leq K)$
$\int_a^K \frac{1}{b - a} dx = P(X \leq K)$

UNIFORM QUESTIONS

X is a uniform distribution with $a = 10$ and $b = 20$. Determine the following:

(a) $P(15)$
$= 0$ (note: singular probabilities of a CRV is always equal to 0).

(b) $P(X \geq 14)$
$X \sim U(10, 20)$
$= \int_{14}^{20} \frac{1}{20-10} dx = 0.6$

(c) $P(X \geq 14 | X \leq 18)$
$= \int_{14}^{18} \frac{1}{20-10} dx / \int_{10}^{18} \frac{1}{20-10} dx = 0.5$

Y is a uniform distribution with $a = 1$ and $b = 5$. Determine the value of k in the following equations:

(a) $P(X > k | X < 3) = 0.5$
$\frac{P(k<X<3)}{P(X<3)} = 0.5$
$P(k < X < 3) = 0.25\ \therefore k = 2$

(b) $P(X > 2 | X < k) = 0.5$
$\frac{P(2<X<k)}{P(X<k)} = 0.5$
$P(2 < X < k) = 0.5P(X < k)$
Using trial and error for values of k:
$k = 3$

NORMAL DISTRIBUTION

ABOUT NORMAL
- Normal distribution has the iconic "Bell Curve" shape which means that data closer to the mean has a higher chance of occurring.
- E.g. finding faults in cars from an entire factory.

NORMAL NOTATION
$X \sim N(\mu, \sigma^2)$ where:
- μ – mean
- σ – S.D.

NORMAL RULES
- $E(X) = \mu$
- $Var(X) = \sigma^2$
- $\sigma = \sigma$

NORMAL CLASSPAD

FIND $P(A \leq X \leq B)$
$normCDF(A, B, \sigma, \mu)$
Where:
- **normCDF** is found in Main App → Action → Distribution → Continuous

FIND K GIVEN $P(X \leq K)$
$invNormCDF("TS", P(X \leq K), \sigma, \mu)$
Where:
- **invNormCDF** is found in Main App → Interactive → Distribution → Inverse

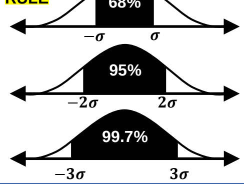
Z-SCORE AND 68 / 95 / 99.7 RULE

Z-SCORE
- $Z \sim N(0,1)$

Where: $Z = \frac{X-\mu}{\sigma}$

Z-Scores simplifies all normal distributions to a mean of 0 and a S.D. of 1. Z-scores indicate how many S.D.'s away from the mean each score is.

68/95/99.7 RULE



PROBABILITY AND RANDOM VARIABLES

NORMAL QUESTIONS

If $X \sim N(\mu, \sigma^2)$ such that the mean is twice the variance and $P(X > 10) = 0.3$. Find μ and σ.

$\mu = 2\sigma^2, \therefore X \sim N(2\sigma^2, \sigma^2)$. *Note: use Z-Scores.*

invNormCDf("L", 0.3,1,0) = 0.5244

$Z = \frac{X-\mu}{\sigma}, 0.5244 = \frac{10-\mu}{\sigma}, 0.5244 = \frac{10-2\sigma^2}{\sigma}$

solve($0.5244 = \frac{10-2\sigma^2}{\sigma}$)

$\sigma = 2.11$ or ~~-2.37~~ and $\mu = 2(2.11)^2 = 8.89$

(a) The time in hours that a brand of light globe operates before going out is normally distributed with a mean of 9000 hours and a standard deviation of 450 hours, what is the probability of a light globe lasting more than 8000 hours given that it does not last more than 10000 hours?

$X \sim N(9000, 450^2) \to P(X \geq 8000 | X \leq 10000) = \frac{P(8000 \leq X \leq 10000)}{P(X \leq 10000)} = 0.9867$

(b) Determine how many life hours are exceeded by 55.6% of all light globes.

$P(X < k) = 0.556 \to invNormCDF(\text{"L"}, 0.556, 450, 9000) = 9063.3759$ hours

QUANTILES

QUANTILE RULES

- $P(X < t_a) = a$ where: $0 < a < 1$
- The a^{th} percentile is the score that a% of the population lies below.

Percentile	Probability
25th	$P(X < t_a) = 0.25$
50th	$P(X < t_a) = 0.50$
75th	$P(X < t_a) = 0.75$

CENTRAL LIMIT THEOREM

CENTRAL LIMIT THEOREM RULES

- Regardless of the original distribution, if the number of independent random samples of the experiment is a large number ($n \geq 25$), the data can be modelled using a normal distribution.

ORIGINAL DISTRIBUTION IS UNKNOWN

- As $n \geq 25$, the distribution becomes normal with the following parameters:
  - Mean stays the same: $\bar{X}$
  - Standard Deviation changes to: $\frac{\sigma}{\sqrt{n}}$

ORIGINAL DISTRIBUTION IS BERNOULLI

- As $n \geq 25$, the distribution becomes normal with the following parameters:
  - Mean stays the same: $\hat{p}$
  - Standard Deviation changes to: $\sqrt{\frac{p(1-p)}{n}}$

SAMPLE NOTATION

- μ and p are statistics of the original population.
- $\bar{X}$ and $\hat{p}$ are statistics of the sample population.

Z-SCORE FOR AN UNKNOWN DISTRIBUTION

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Z-SCORE FOR A BERNOULLI DISTRIBUTION

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

NATURE AS N CONTINUES TO INCREASE

- As $n \to \infty$, the distribution approaches the standard normal distribution.

CONTUNUITY CORRECTION

CONTINUITY CORRECTION RULES

- When using a discrete distribution and $n \geq 25$, it can be modelled using a normal distribution (continuous).
- When changing from a discrete distribution to a continuous distribution, the probabilities you calculate change slightly according to the table:

Discrete	Continuous
$P(X = k)$	$P(k - 0.5 < X < k + 0.5)$
$P(X > k)$	$P(X > k + 0.5)$
$P(X \geq k)$	$P(X > k - 0.5)$
$P(X < k)$	$P(X < k - 0.5)$
$P(X \leq k)$	$P(X < k + 0.5)$

CENTRAL LIMIT THEOREM QUESTIONS

(a) In one particular store, 18% of pizzas are overcooked. In a sample of 150 pizzas, describe the distribution and state the mean and standard deviation.

$p = 0.18$ and $s = \sqrt{\frac{0.18(1-0.18)}{150}} = 0.0314$

Hence, $\hat{p} \sim N(0.18, 0.0314^2)$

(b) Determine the probability that a point estimate for the proportion of overcooked pizzas exceeds 0.21.

$P(\hat{p} > 0.21) = 0.1697$

(a) 23% of Australians are left handed. If a sample of 40 Australians are surveyed, what proportion of these samples are expected to contain less than 20% of left-handers?

$\hat{p} \sim N(p, s^2)$ where $p = 0.23$ and $s = \sqrt{\frac{0.23(1-0.23)}{40}}$

$\hat{p} \sim N(0.23, 0.0665^2)$ and $P(\hat{p} < 0.2) = 0.3257$

(b) What proportion of these samples are expected to contain between 10% and 15% of left-handers?

$P(0.10 < \hat{p} < 0.15) = 0.0892$

INTERVAL ESTIMATES

CONFIDENCE INTERVALS AND MARGIN OF ERROR

CONFIDENCE INTERVAL RULES

- $\left(\hat{p} - Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$
- $\hat{p} \pm E$

Where:

- $Z \to$ Z-Score for a given confidence interval (refer to table below for common z-scores):

% Confidence Interval	Z-Score
99% CI	2.58
95% CI	1.96
90% CI	1.645

Custom confidence interval:

$z_c = -1\times$ invNormCDf("C", c, 1, 0)

Where:

$c \to$ CI% as a decimal

MARGIN OF ERROR RULES

- $E = Z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- $E \propto \frac{1}{\sqrt{n}}$
- Calculate the width of a confidence interval: $Width = 2E$

CHANGING CONFIDENCE INTERVALS

If you are changing the confidence interval of a question (i.e. you are given a 95% CI and you need to determine a 99% CI), follow these steps:

Step 1: determine p:

$p = \frac{lower\ bound + upper\ bound}{2}$

Step 2: determine E: $E = upper\ bound - p$

Step 3: determine E_{new}: $E_{new} = \frac{z_{new}}{z_{old}} \times E$

Step 4: determine new confidence interval:

$New\ CI = p \pm E_{new}$

CONFIDENCE INTERVALS AND MARGIN OF ERROR QUESTIONS

A 90% Confidence Interval is (0.38, 0.45). Determine a 95% Confidence interval.

$p = \frac{0.38 + 0.45}{2} = 0.415$

$E = 0.45 - 0.415 = 0.035$

$E_{new} = \frac{1.96}{1.645} \times E = 0.0417$

$95\%\ CI = 0.415 \pm 0.0417$

$= (0.3733, 0.4567)$

How many times larger is the margin of error of a sample of 1225 compared to a sample of 11025?

$E \propto \frac{1}{\sqrt{1225}} = \frac{1}{35}$

$E \propto \frac{1}{\sqrt{11025}} = \frac{1}{105}$

$\therefore$ 3 times as large.

(a) In a random sample of 400 people, 129 we male. Calculate a 90% confidence interval.

$\hat{p} = \frac{129}{400} = 0.3225$

$90\%\ CI = 0.3225 \pm 1.645\sqrt{\frac{0.3225(1-0.3225)}{400}}$

$90\%\ CI = (0.2898, 0.3552)$

(b) What is the largest size sample of people that would have to be taken in order for a width of a 99% confidence interval to be 0.1 or less?

$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \to \frac{0.1}{2} = \sqrt{\frac{0.3225(1-0.3225)}{n}}$

Solving for n: $n = 87.3975 \approx 88$ people are needed.

What is the margin of error on a 99% confidence interval of (0.25, 0.32)?

$E = \frac{0.32 - 0.25}{2} = 0.035$

Note: $\hat{p}$ does not have to be given to determine the margin of error.

YOUR NOTES AND EXAMPLES